



Schrödinger-Ashutosh Oscillations

Ashutosh Sharma¹, Vinit V Shenvi²

¹Student, M.S Ramaiah Institute of Technology, Bengaluru, Karnataka, India

²Student, M.S Ramaiah Institute of Technology, Bengaluru, Karnataka, India

Corresponding Author: Ashutosh Sharma

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ABSTRACT: The study of interatomic interactions has been one the most pertinent endeavors in science – its expositions and theories are progressively profound and comprehensive. This theoretical physics paper aims at probing into the nature of oscillations at the quantum level and discerning the fact that it could contentedly explain these electron transitions and their wave-particle behavior which is quite contrary to classical oscillations occurring within the purview of classical mechanics.

KEYWORDS: Oscillation, Energy, Schrödinger Equation, Wave Function, Orbital Transition.

I. INTRODUCTION

In this paper, we consider the quantum mechanical model of the atom as the basis for our calculations. Our approach is based on the idea established by Louis de Broglie in 1920 that matter has both particle and wave nature. It is thereby also known that at the subatomic level, the dual-nature exhibition of particles is much more significant than larger bodies.

Here, we consider a free electron oscillating in space-time. Clearly, it transpires from the potential energy force exerted by the nucleus on the electron.

In quantum mechanics, the equation of motion of the subatomic particle is described by the time-dependent Schrödinger Equation which determines the wave function at a given time. In the system considered here, which has a definite total energy, the wave function is essentially a standing wave. Thus, it can be said that for an electron bound to oscillation, the standing wave equations hold true. Their corresponding energies can be deduced from their respective wave functions by incorporating the time-dependent Schrödinger Equation in its principality.

The general notation $\psi(x, t)$ would be used to represent the wave function, which, in this case is a function of both position and time. The superposition principle of waves warrants that the sum of two waves itself is a possible wave motion. The possible wavelengths of a standing wave are quantized, which helps us assert the boundary conditions that lead to the quantization for quantum waves.

At different instances in the numerical proposition concerning waves, they have been made more adequately generalized. The attribute of quantum mechanics that in it, the wave function can be a complex number and that quantum standing waves are complex, is considered. Any quantum system that has definite energy will have a wave function of the forms obtained in following sections.

It is important to note that these electronic oscillations intrinsically occur within the limits of a particular electron orbital. This assumption shall be proven further and is crucial in displaying the validity of this theory, because unbounded oscillations would not only render the atom unstable (which is not true), but also falsify the quantum mechanical model. Notably, the total energy of the atomic system is definite and conserved. Had it not been, the atom and the universe would be inherently unstable, which is naturally not the case.

These oscillations are unique and explain electronic transitions, to which we assign the name of 'Schrödinger-Ashutosh Oscillations'. The verity of the theory is validated if it is deemed sufficiently consistent with accepted theories that explain these transitions.

II. CALCULATIONS

Notations used:

ψ – General wave function

Ψ – Wave function of the electron

E – Total energy of the electron

U – Potential energy of the electron

\mathcal{K} – Kinetic energy of the electron



\mathcal{V} - General notation of Potential Energy
 \hbar - Reduced Planck's Constant
 h - Planck's Constant
 m - Mass of the electron
 x - Position of the electron
 t - Time
 k - Wave function corresponding to electron
 ω - Angular velocity corresponding to electron
 A, B, a, b - Amplitude denotations
 λ - Wavelength of the oscillating electron
 ϕ - Arbitrary Constant
 $i = \sqrt{-1}$
 n - An arbitrary quantized positive integer for standing waves

In general, for two sinusoidal travelling waves moving in opposite direction,

$$\begin{aligned}\psi_1(x, t) &= B\sin(kx - \omega t) \\ \psi_2(x, t) &= B\sin(kx + \omega t)\end{aligned}$$

Their superposition itself is a wave motion:

$$\begin{aligned}\psi(x, t) &= B[\sin(kx - \omega t) + \sin(kx + \omega t)] \\ \Rightarrow \psi(x, t) &= 2B\sin kx \cos \omega t\end{aligned}$$

Setting $2B = A$, $\psi(x, t) = A\sin kx \cos \omega t$

Where the spatial function $\Psi(x) = A\sin kx$

But $\Psi(x)$ can be a more complicated function where time dependence is still sinusoidal, thus it can be a sine or a cosine function. In a more general format,

$$\psi(x, t) = \Psi(x)(a\cos\omega t + b\sin\omega t) \forall a, b \in \mathbb{R}$$

However, the quantum interactions may express themselves in complex planes.

Therefore,

$$\Psi(x, t) = \Psi(x)(\cos\omega t - i\sin\omega t)$$

Using Euler Formula $e^{i\theta} = \cos\theta + i\sin\theta$,

$$\Rightarrow \Psi(x, t) = \Psi(x)e^{-i\omega t}$$

Now on applying the standing wave condition,

$$A = \frac{n\lambda}{2} \quad \forall n \in \mathbb{N}$$

Since, $k = \frac{2\pi}{\lambda}$, $A = \frac{n\pi}{k}$,

$$\Psi(x, t) = \frac{n\pi}{k} \sin kx e^{-i\omega t}$$

The probability density is represented by,

$$\begin{aligned}|\Psi(x, t)|^2 &= |\Psi(x)|^2 |e^{-i\omega t}|^2 = |\Psi(x)|^2 \\ &= A^2 \sin^2 kx\end{aligned}$$

For $x = \frac{\lambda}{4}$,

$$\begin{aligned}\Psi(x, t) &= \frac{n\pi}{2} e^{-i\omega t} \\ \Rightarrow |\Psi(x, t)|^2 &= A^2\end{aligned}$$

Which denotes the maximum probability density. Since it is a rational number, our equations are well within the constraints that define the nature of atomic orbitals.

Now, to determine the behavior of these waves, we shall ascertain the kinetic and potential energy of these electrons, both of which are functions of position(x) and time (t).

The Schrödinger Equation indeed is

$$\hat{H}\Psi = E\Psi$$

Which can also be expressed as

$$\frac{-\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + \mathcal{V}(x, t)\Psi = E\Psi$$

Using $\hbar = \frac{h}{2\pi}$,

$$\frac{-h^2}{8\pi^2 m} \frac{\partial^2 \Psi}{\partial x^2} + (\mathcal{V}\Psi) = E\Psi$$

Where $\frac{-h^2}{8\pi^2 m} \frac{\partial^2 \Psi}{\partial x^2}$ is the kinetic energy part of the equation.

Now,

$$\begin{aligned}\frac{\partial^2 \Psi}{\partial x^2} &= \frac{-Ak}{2} [\sin(kx + \omega t)(k + \omega) \\ &\quad + \sin(kx - \omega t)(k - \omega)]\end{aligned}$$

Therefore, Kinetic Energy

$$\begin{aligned}\mathcal{K}(x, t) &= \frac{h^2 k A}{16\pi^2 m} [\sin(kx + \omega t)(k + \omega) \\ &\quad + \sin(kx - \omega t)(k - \omega)]\end{aligned}$$

At $t = 0$,

$$\mathcal{K} = \frac{h^2 k A \omega}{8\pi^2 m} \sin \omega t$$

Using Dispersion Relation $\omega = \frac{hk^2}{2m}$,



$$\mathcal{K} = \frac{h^3 k^3 A}{32\pi^3 m^2} \sin \omega t$$

At $t = \phi\pi, \phi \in \mathbb{N} \quad t = 0, \pi, \dots, \phi\pi, \quad \mathcal{K}_{\min} = 0$

Now for Potential Energy,

Considering the equation

$$\Psi(x, t) = \frac{n\pi}{k} \sin kx e^{-i\omega t},$$

Velocity,

$$v = \frac{\partial \Psi}{\partial t} = -i\omega \frac{n\pi}{k} \sin kx e^{-i\omega t}$$

Force,

$$F = m \frac{\partial v}{\partial t} = -m\omega^2 \frac{n\pi}{k} \sin kx e^{-i\omega t}$$

Potential Energy,

$$\mathcal{U} = -\frac{\partial F}{\partial x} = m\omega^2 n\pi e^{-i\omega t} \cos kx$$

At $x = 0, \quad |\mathcal{U}_{\max}| = m\omega^2 n\pi e^{-i\omega t}$

For $t = 2\phi\pi, \phi \in \mathbb{N}, \mathcal{U}_{\min} = m\omega^2 n\pi$

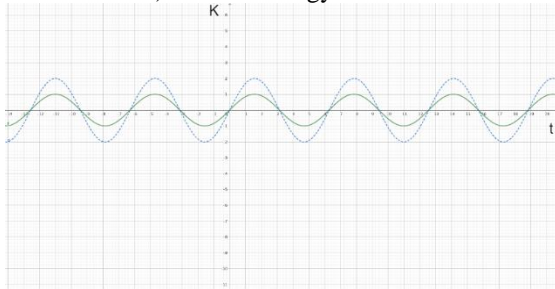
(Here the electron tends to become unbounded or free)

For

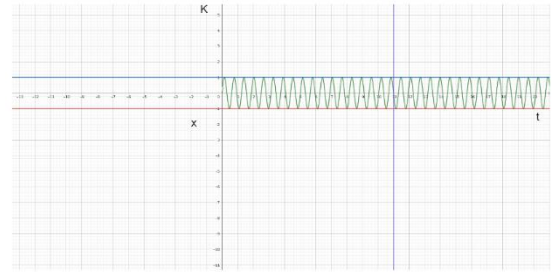
$$t = (2\phi + 1)\pi, \phi \in \mathbb{N}, \mathcal{U}_{\max} = -m\omega^2 n\pi$$

III. GRAPHS

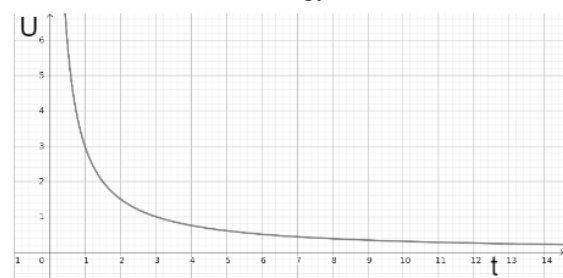
A) Kinetic Energy vs Time



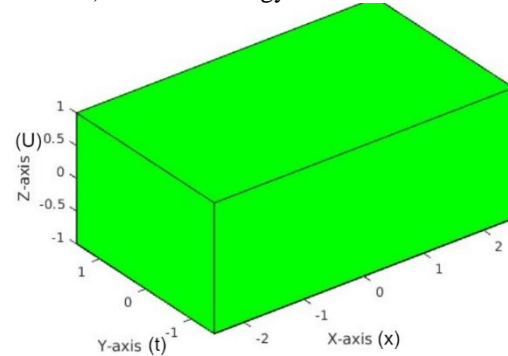
B) Kinetic Energy vs Time vs Position



C) Potential Energy vs Time



D) Potential Energy vs Time vs Position



IV. CONCLUSIONS

The inferences that can be drawn from the kinetic energy relations are:

- Kinetic Energy of a subatomic particle is always expressed in the real plane and is always sinusoidal.
- The Kinetic Energy is appreciably substantial at the mean position of oscillation except when $t = 0$, at which the kinetic energy is minimum. The Kinetic Energy is also maximum at $x = \frac{\lambda}{4}$
- The Kinetic Energy variation can be visualized as an infinitely continuous three-dimensional tunnel.



The inferences that can be drawn from the potential energy relations are:

- a) At the mean position of oscillation, the electron can have either maximum or minimum potential energy.
- b) Potential Energy expresses itself in the complex plane as well. It exponentially decreases with time.
- c) Quantum oscillations are very different from physical oscillations such as those exhibited by the pendulum.
- d) Quantum oscillations are special damped oscillations essentially occurring in the absence of a well-defined damping medium.
- e) The Potential Energy variation can be visualized as an infinitely continuous cuboid containing sinusoidal waves that tend to suddenly vanish with time.
- f) However, as the total energy of any system must remain conserved, the energy lost as a result of the potential energy vanishing over time, must be equivalent to the transition energy.
- g) Therefore, the electron now behaves like a particle which carries the lost potential energy as hc/λ
- h) As orbital energy levels extend infinitely into space-time, when the electrons tend to experience the lower energy state, tend to find the immediate relatively higher energy state.

Thereby, these oscillations competently explain interatomic interactions and are consistent with accepted theories that explain these transitions. Further, they can also explain orbital overlapping in case of metals, which by nature, possess abundant free electrons i.e., they have an inherent tendency of performing higher oscillations.

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